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PROBLEM OF THE PALAEO-WARTA RIVER VALLEY ON THE WOŹNIKI-WIELUŃ UPLAND IN THE LIGHT OF GEOSTATISTICAL SPATIAL ANALYSIS OF THE SUB-PLEISTOCENE SURFACE HYPSOMETRY

Key words:

geostatistics, kriging, palaeogeomorphology, elevation model, sub-Pleistocene hypsometry, palaeo-Warta River valley, Woźniki-Wieluń Upland

Abstract

Digital elevation model of the sub-Pleistocene surface on the Woźniki-Wieluń Upland was constructed using ordinary point kriging with the aim of verifying the existence of the buried valley of the palaeo-Warta River. The problem of its existence, often dealt with in literature, is of key importance in the restoration of the palaeogeographic evolution of the northern part of the Silesia-Kraków Upland. The application of a geostatistical model appropriate for the spatial structure of the data allowed to elaborate a digital elevation model of the buried surface, which seems to be the most accurate representation of palaeorelief. The models reveals the depressions described in literature. However, their pattern does not support the thesis that they are fragments of a large valley. Their shapes are characteristic of subglacial kettles, troughs and basins.

PROBLEM DOLINY PRA-WARTY NA WYŻYNIE WOŹNICKO-WIELUŃSKIEJ W ŚWIETLE GEOSTATYSTYCZNEJ ANALIZY PRZESTRZENNEJ HIPSOMETRII POWIERZCHNI PODPLEJSTOCEŃSKIEJ

Słowa kluczowe:

geostatystyka, kriging, paleogeomorfologia, model wysokościowy, hipsometria podplejstoceńska, dolina pra-Warty, Wyżyna Woźnicko-Wieluńska

Abstract

Metodą geostatystyczną – krigingiem zwyczajnym punktowym opracowano cyfrowy model wysokościowy powierzchni podplejstoceńskiej na Wyżynie Woźnicko-Wieluńskiej, w celu rozsądzenia, czy istnieje kopalna dolina pra-Warty. Problem ten, wciąż dyskutowany w literaturze, jest zarazem kluczowym w opracowaniu koncepcji rozwoju paleogeograficznego północnej części Wyżyny Śląsko-Krakowskiej. Zastosowanie modelu geostatystycznego właściwego dla struktury przestrzennej danych pozwoliło opracować cyfrowy model wysokościowy kopalnej powierzchni, który

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z dużym prawdopodobieństwem obrazuje paleohipsometri. Widoczne są na nim obniżenia opisywane w literaturze. Jednak ich układ nie jest potwierdzeniem tezy, że są to fragmenty dużej doliny. Kształtem odpowiadają one kotłom, rynnom i misom subglacjalnym.

Introduction

A buried valley of the palaeo-Warta River on the Woźniki-Wieluń Upland has been repeatedly described in geological and geomorphological publications as one of the major forms of pre-Pleistocene relief. However, no convincing evidence of its existence has been hitherto presented, as its course was reconstructed basing on various depressions found at various, often distant, locations. The lack of evidence for their intercommunication raises doubts about the existence of a buried pre-glacial valley of the Palaeo-Warta River in the sub-Pleistocene relief of the Woźniki-Wieluń Upland.

Geostatistical methods allow for the development of a credible digital elevation model of the sub-Pleistocene surface for a large area defined by the extent of boreholes and exposures of sub-Quaternary rocks. Such a model would allow for a unanimous interpretation of the buried depressions.

The aim of this paper is to present the results of an analysis of a digital elevation model of the sub-Pleistocene surface in the central part of the Woźniki-Wieluń Upland, constructed using geostatistical methods.

The digital elevation model has been constructed using all available data from borehole archival data and geological maps and it covers the whole area defined by the available data. It is also made as accurate as possible in order to make the palaeogeomorphological interpretation reliable.

The Woźniki-Wieluń Upland is the north-western part of the Silesia-Kraków Upland. It borders the Silesian Upland on the west and the Kraków-Częstochowa Upland on the south (Fig. 1). The study was done in the Górna Warta Depression and the Krzepice Depression, between Częstochowa and Krzepice. This fragment of the Upland was densely drilled for iron ore prospection by Przedsiębiorstwo Geologiczne in Częstochowa.

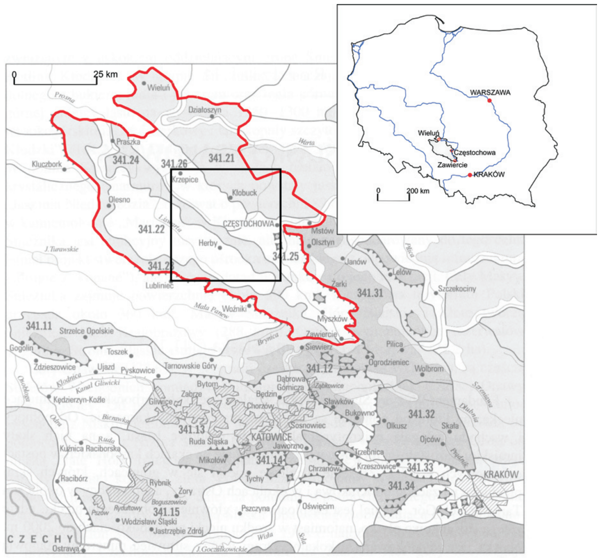


Fig. 1. Location of the study area with respect to the mezoregions of the Woźniki-Wieluń Upland according to J. Kondracki (2002). Mezoregions within the Woźniki-Wieluń Upland: 341.21 –Wieluń Upland, 341.22 – Liswarta-Prosna Depression, 341.23 – Woźniki Cuesta, 341.24 –Herby Cuesta, 341.25 Górna Warta Depression, 341.26 –Krzepice Depression.

Ryc. 1. Położenie terenu badań na tle mezoregionów Wyżyny Woźnicko-Wieluńskiej wg J. Kondrackiego (2002). Mezoregiony wydzielone w obrębie Wyżyny Woźnicko-Wieluńskiej: 341.21 – Wyżyna Wieluńska, 341.22 – Obniżenie Liswarty-Prosny, 341.23 – Próg Woźnicki, 341.24 – Próg Herbski, 341.25 Obniżenie Górnej Warty, 341.26 – Obniżenie Krzepickie.

Methods

The digital elevation model was constructed using ordinary point kriging. It is based on values of altitudes of Pleistocene base at sample points. The values are regionalized. Each of the values is a realization of a random function. In such a context, the geostatistical method is optimal for data manipulation.

In geostatistical (stochastic) approach, altitude, as a regionalized variable, has two aspects: random and structural. The random aspect accounts for local irregularities, while the structural one accounts for the large-scale tendencies (Matheron, 1989). Randomness of the analyzed variable is represented by fluctuations around the fixed surface – drift. The fluctuations are vanishing properties of the studied phenomenon, featuring their own structure (Wackernagel, 2003).

The geostatistical analysis of elevation of the sub-Pleistocene surface was performed in two stages. The first was geostatistical analysis of data, the second was spatial analysis (interpolation) using ordinary point kriging.

Geostatistical analysis

The aim of the geostatistical (structural) analysis was to present the spatial structure of the data set variability using an empirical semivariogram, and then to construct a geostatistical model of this structure. The analysis was preceded by the characterization of the source data and their statistical distribution.

Source data and their evaluation

The source data ˜ altitudes of the sub-Pleistocene surface were obtained for 6499 sample points. Of these, 4862 values were calculated using borehole data and 1637 were read from the Detailed Geological Map of Poland, within the outcrops of sub-Pleistocene strata (Bardziński et al., 1982, Bednarek et al., 1987, Haisig et al., 1981a, 1981b, 1985, Kaziuk et al., 1986).

Statistical distribution of the sample data set is not favourable for geostatistical analysis because of the right-handed asymmetry, high concentration of values and high variability of data (Table 1).

Table 1. Parameters of statistical distribution of the data set
Tabela 1. Parametry rozkładu statystycznego zbioru danych

Frequency	Mean	Min.	Median	Max.	Standard deviation	Variance	Skewness coefficient	Curtosis coefficient
6499	260.35	166.58	260.99	365.53	22.98	528.12	0.0505	1.77

The sample points are unevenly distributed over an area of 1540 km^2. A distinct concentration is present in an NW-SE oriented belt, several kilometres wide, between Poraj, Częstochowa, Kłobuck and Krzepice (Fig. 2). It corresponds to the subcrop of iron ore-bearing Middle Jurassic clays to the sub-Pleistocene surface. The density of the sample points is variable. It is greatest within ancient mining fields, 78 points per 1 km^2 at Wręczyca. One hundred seventy points of the kilometric grid contain only one sample point and 770 fields contain none. On average, there are 4.2 points per 1 km^2. Distances between points vary from ca. 1 km to ca. 50 m.

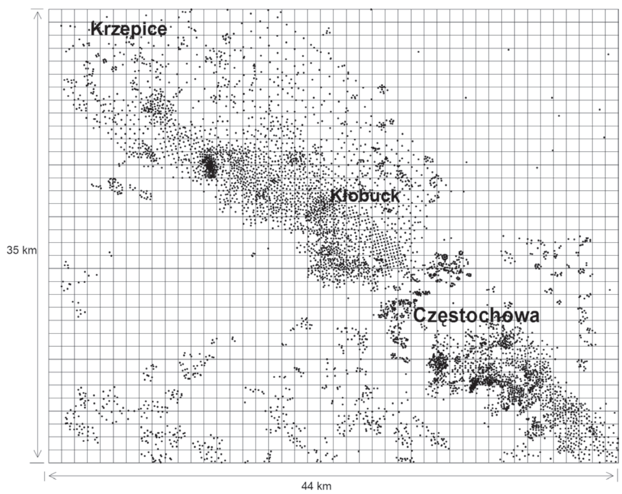


Fig. 2. Areal distribution of the sample points
Ryc. 2. Rozmieszczenie punktów próbkowych

The accumulated material was evaluated at the stage of data accumulation. The evaluation involved first the correctness of borehole locations, then the correctness of identification of the base Pleistocene sediments in the borehole sections.

The degrees of detail in lithological descriptions of the Pleistocene and Jurassic rocks are comparable. The depths down to the soles of penetrated strata in the whole sections were determined to an accuracy of 1 cm. The base of the Pleistocene sediments is easy to identify in almost all boreholes because they overlie limestones (Upper Jurassic), sandstones (Middle Jurassic) or firm dark micaceous clays, locally even black, with fauna.

These clays, known as „ore-bearing clays" (Middle Jurassic) contain deposits of iron ores. The upper ore horizon lies a few to several metres below the top of the clays.

De identyfication of the Pleistocene clays directly overlying Middle Jurassic clays is not difficult, because the Pleistocene clays are lighter-coloured than the Middle Jurassic ones, gray or light-gray, often varved, and include an admixture of quartz sand or gravel and pebbles of chert or limestone, often also northern (Scandinavian) rocks.

The correctness of identification of the Pleistocene base raised doubts in only few tens (ca 30) boreholes. The Middle Jurassic clays are overlain there by mixed Middle Jurassic and Pleistocene clays. The position of the Pleistocene base in such cases was identified by comparing sections in neighbouring boreholes and drawing a geological cross-section using them. When the interpretation was considered erroneous, a correction was introduced.

Borehole data are valuable evidence, suitable as a base for constructing a reliable digital elevation model of the buried sub-Pleistocene surface.

Calculation of isotropic empirical semivariogram

The spatial structure of altitude of the sub-Pleistocene surface is shown on an empirical isotropic semivariogram.

The measure of the variability in the spatial structure is provided by semivariance calculated for the semivariogram classes as a half of the mean of squares of the deviations of the studied parametre, in points distant by vector *h*

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} (x_i - y_i)^2$$

where:

N(h) – sample frequency,

x_i, y_i – values of the studied parameter (at the start and the end, respectively).

Parameters of the empirical isotropic semivariogram of the altitude values for the sub-Pleistocene surface are shown in Table 2.

Table 2. Main characteristics of the empirical isotropic semivariogram of the altitude values for the sub-Pleistocene surface
Tabela 2. Podstawowe charakterystyki izotropowego semiwariogramu empirycznego wysokości bezwzględnej powierzchni podczwartorzędowej

Geometry of the semivariogram grid	Frequency of the data set	Number of variogram classes	Class range [m]	Number of point pairs
Grid radius: 23 000 m Number of direction classes: 180 Number of distance classes: 100	6499	25	1000	14 841 515

Empirical isotropic semivariogram of the sub-Pleistocene surface elevation (Fig. 3) starts with a distinct discontinuity. At *h=0 γ(h)* it attains the value of 36 m^2. A rapid rise in values of the semivariance function in successive distance classes follows up to the distance of 4100 m. The log is nearly rectilinear and steep over this distance. Farther, up to 7000 m, the value of the function becomes stable at 340 m^2. Over the distance of 7000 – 11,000 m the function value rises again, initially, to 9000 m, first slowly, then more intensely.

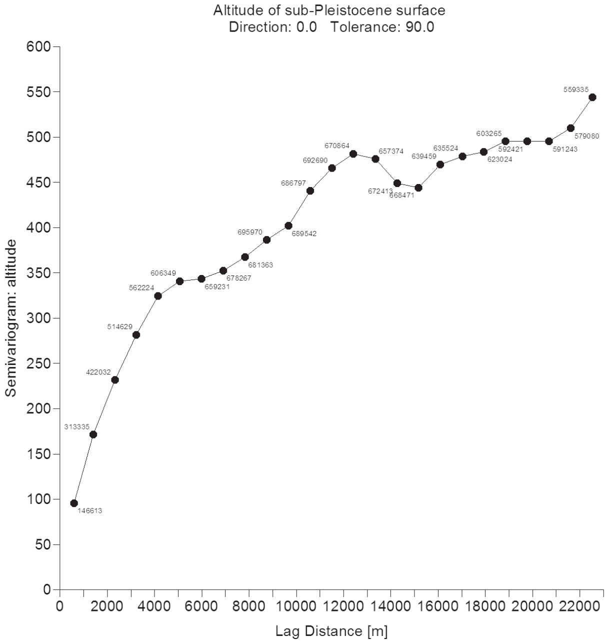


Fig. 3. Empirical isotropic semivariogram of the sub-Pleistocene surface elevation

Ryc. 3. Izotropowy semiwariogram empiryczny wartości wysokości bezwzględnej powierzchni podplejstoceńskiej

Within the distance range of 14,000 – 16,000 m the values of the semivariance function decrease to 440 m^2. They slightly rise again in the final section of the semivariogram, to 21,000 m, with two notable levels of stabilization -– at 460 m^2 and 480 m^2. It rises to 550 m^2 in the two final distance sections of the semivariogram.

The empirical isotropic semivariogram shows the complex structure of the sub-Pleistocene surface elevation values. The structure involves:

– nugget variance (nugget effect) that equals 36 m^2 at h=0, which may partly result from measurement errors caused by errors in determination of the thickness of Pleistocene sediments, which resulted in errors of Pleistocene base elevations;

– distinct spatial correlation of data at distances between samples 1500m – 4200 m, 7000 m – 11,600 m and greater than 20,400 m;

– stabilization of the semivariogram function values, expressed in flattening of the diagram at various, successively greater, values of the semivariance function, if the distance between samples equals 5000–6000 m, 12,000–13,100 m and 18,600–24,000 m;

– reversal of the growing trend of the semivariance function at the distances between samples equal to 13,100–15,000m.

The semivariogram is not smooth; it displays fluctuations. It also has no distinct flattening over a large distance. In general, the values of the semivariance function increase with increasing distance, with one exception when this trend is reversed. This is an unlimited semivariogram (Namysłowska-Wilczyńska, 2006). The persistent increase in semivariogram values with distance is interpreted as indicative of the presence of a trend or drift in the analyzed data.

Geostatistical model of variability

The next step in the geostatistical analysis of the empirical semivariogram involved approximation to an analytical function, which is a geostatistical model of the variance of studied parameter (Mucha, 1994), called also the theoretical model of the semivariogram or the variance structure model (Fig. 4).

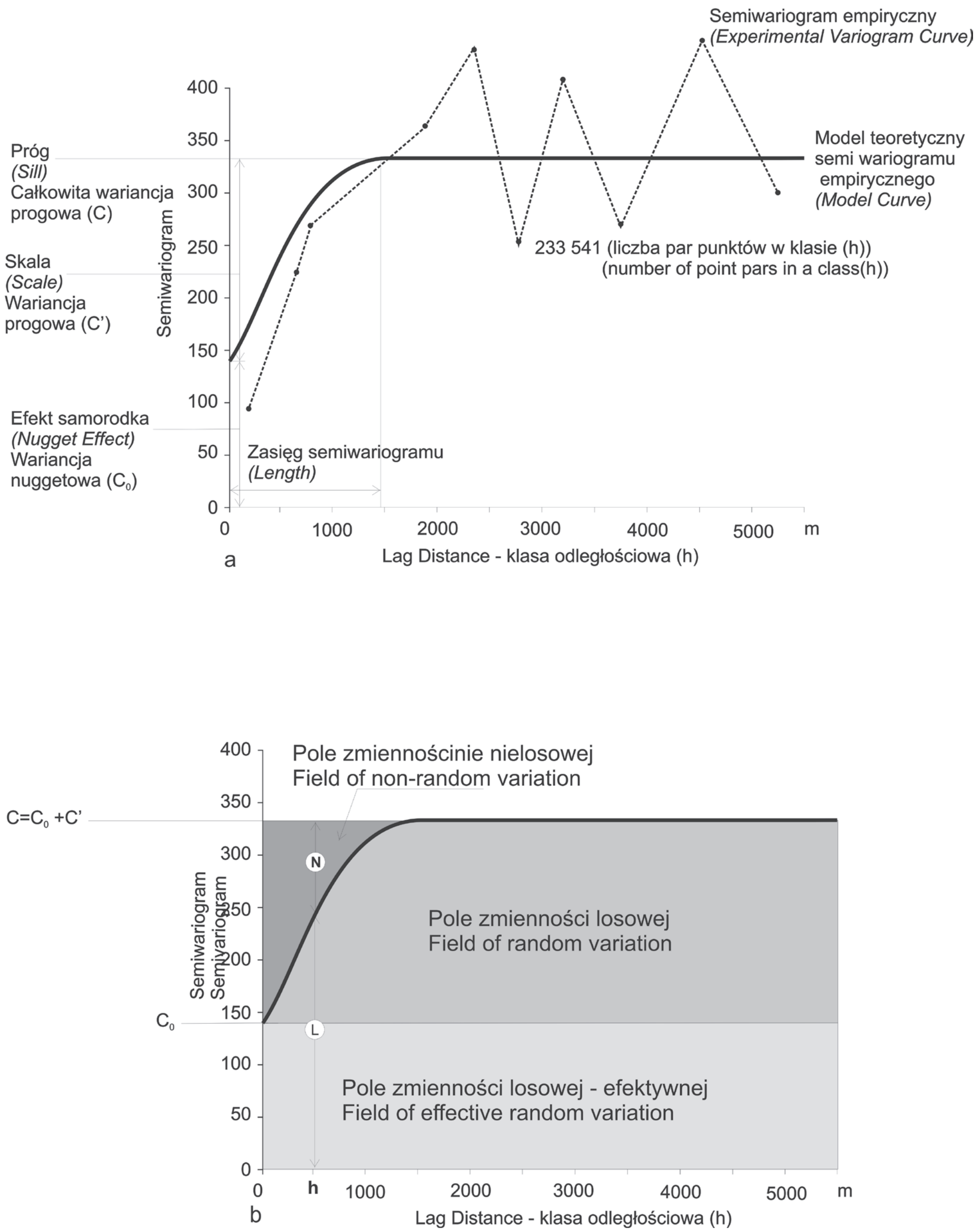


Fig. 4. Experimental semivariogram fitted using a theoretical model (a) (*Surfer 8*); variance structure of the studied parameter described by exemplary theoretical semivariogram (geostatistical model) after J. Mucha, (2002): *C* – total sill variance (semivariogram amplitude), C_0 –nugget variance (local variability of the parameter), *C'* – sill variance, *L*, *N* – random and non-random components of the studied parameter for distance *h* between sample points.

Ryc. 4. Semiwariogram empiryczny dopasowany za pomocą modelu teoretycznego (a) (*Surfer* 8); struktura zróżnicowania badanego parametru opisana przykładowym semiwariogramem teoretycznym (modelem geostatystycznym) za J. Mucha, (2002): *C* – całkowita wariancja progowa (amplituda semiwariogramu), C_0 – wariancja nuggetowa (zmienność lokalna parametru), *C'* – wariancja progowa, *L*, *N* – losowy i nielosowy składnik badanego parametru dla odległości, *h* między punktami próbkowymi.

After unsuccessful attempts at approximating the empirical semivariogram with single basic models, complex embedded models were used. These combined nugget effect models with one, two or three basic models (Table 3).

Table 3. Chart of parametric values of the geostatistical elevation models of the sub-Pleistocene surface
Tabela 3. Zestawienie wartości parametrów modeli geostatystycznych wysokości bezwzględnej powierzchni podplejstoceńskiej

Semivariogram model	Nugget effect (C_0) (Nugget variance) (Nugget Effect) [m a.s.l.]^2	Root of nugget variance $\sqrt{C_0}$ [m a.s.l.]	Scale (C') (Sill variance) (Scale) [m a.s.l.]^2	Root of sill variance $\sqrt{C_0}$ m a.s.l.	Sill $C=C_0+C'$ (Total sill variance (Sill) [m a.s.l.]^2	Tangent of slope of linear function (Slope)	Exponent of power function (Power)
Model (a): Nugget + linear	252	15,87				0,0129	
Model (b): Nugget + power	44,9	6,70	0,768	0,876	45,67		0,635
Model (c): Nugget + exponential + power	23,5	4,85	37,2 0,73	6,099 0,854	61,43		0,635
Model (d): Nugget + spherical + power	42,8	6,54	27,9 0,694	5,282 0,833	71,39		0,635
Model (e): Nugget + pentaspherical + power	65,36	8,08	9,2 0,847	3,033 0,92	75,41		0,625

Geostatistical modelling began with an approximation of the experimental model by a model consisting of a linear function (default model in GRID module of SURFER software) and a nugget model (Fig. 5a). Satisfactory results were obtained in successive attempts by combination of the nugget model with a power function (Fig. 5b) or with several basic models. The best fit of the theoretical and experimental models was obtained when the power function was one of the elements of the composite model (Fig. 5c–e).

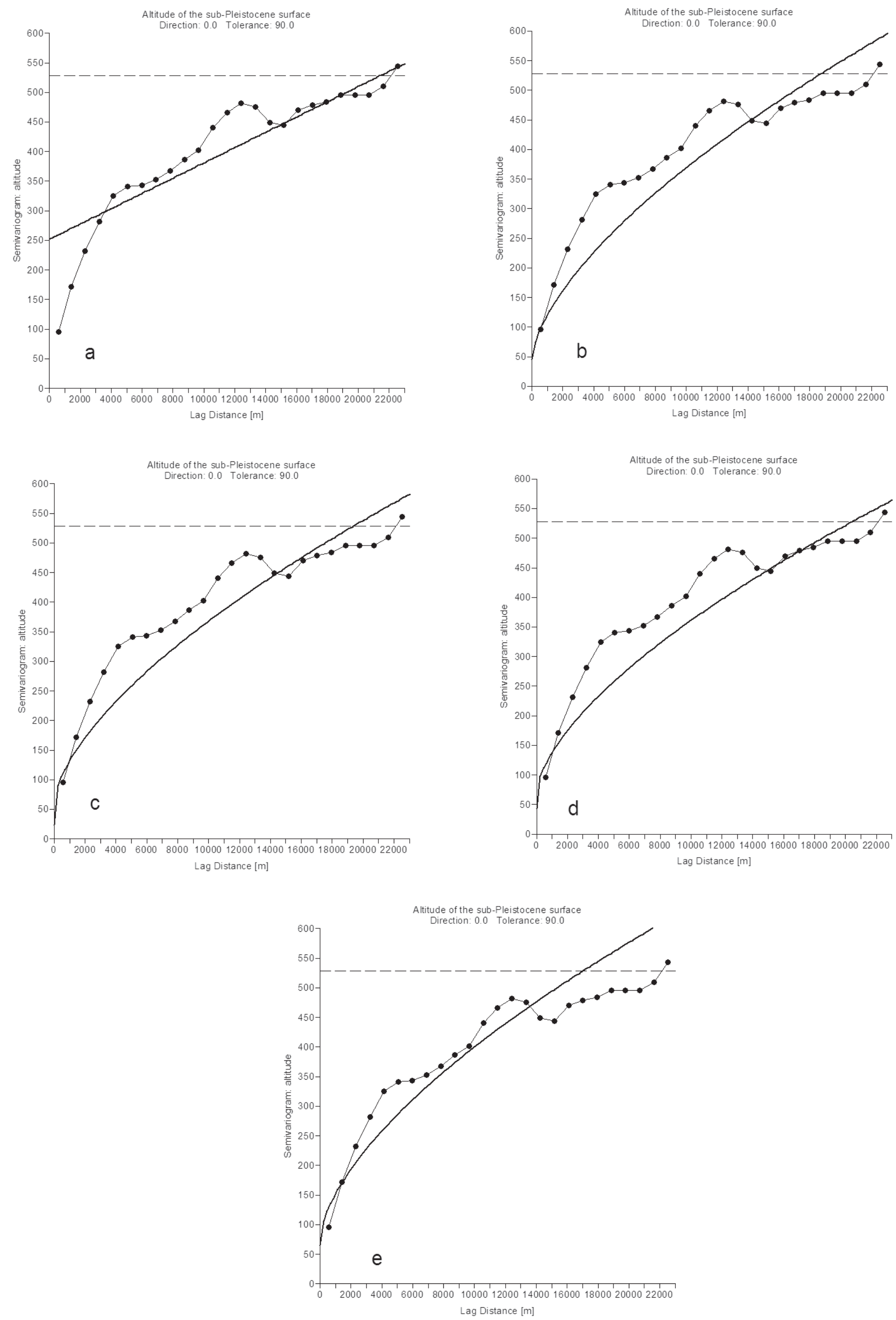


Fig. 5. Geostatistical elevation models of the sub-Pleistocene surface
a – nugget + linear model, b – nugget + power model, c – nugget + exponential + power model, d – nugget + spherical + power model, e – nugget + pentaspherical + power model.

Ryc. 5. Modele geostatystyczne wysokości bezwzględnej powierzchni podplejstoceńskiej
a – nugget effect (efektu bryłki) i liniowy, b – nugget effect i power (potęgowy), c – nugget effect, exponential (wykładniczy) i power, d – nugget effect, spherical (sferyczny) i power, e – nugget effect, pentaspherical i power.

Noteworthy in the tested models (except for model c) is the high value of the nugget variance – C_0 (random component of the variability) at h=0 (Table 3). It is greatest in model a, thus precluding its use in spatial analysis. The exponent of the power function in models b, c and d is the same; it is much lower in model e.

Cross-validation of the estimation and validation of the interpolation

The validation of the quality of estimation and interpolation was performed with the aim of selecting the best model of data variability for optimal interpolation of the sub-Quaternary surface elevation. The quality of fit of the theoretical model to the experimental one has a direct influence on the accuracy of the estimation and interpolation. This in turn influences the degree of conformance of the digital model with reality.

The validation was done in two steps. First the theoretical models were validated using cross-validation, then the quality of interpolation performed was assessed using the successive models.

The process of cross-validation consisted in the elimination of successive sample points $Z(x_\alpha)$ from the data set and in the estimation of $Z^*(x_{[\alpha]})$ values at the location of the eliminated sample point using the remaining data (Issaks and Srivastava, 1989). Then what followed was the calculation of the cross-validation error (true estimation error), that is the difference between the original value and the mean estimated value, showing to what degree the value of data fits the neighbourhood of surrounding data values (Namysłowska-Wilczyńska, 2006):

$$Z(x_\alpha) - Z^*(x_{[\alpha]})$$

where:

$Z(x_\alpha)$ – original value

$Z^*(_{[\alpha]})$ – mean value estimated at location x_α calculated after the elimination of the original valule.

The validation of the theoretical models of the semivariographs took into account: mean of cross-validation errors (ME), root of mean of square cross-validation errors (RMSE), standard deviation of kriging ($\sigma_{[\alpha]}$), kriging variance (σ_k^2) and coefficient of linear correlation (r) between data in sample points $Z(x_\alpha)$, and mean estimated values in these points $Z(x_{[\alpha]})$ (Table 4).

Table 4. Results of cross-validation of the estimation of elevation of the sub-Pleistocene surface
Tabela 4. Wyniki kross-walidacji estymacji wysokości bezwzględnej powierzchni podplejstoceńskiej

Estimation with semivariance model	Mean estimation error ME [m a.s.l.]	Mean square of estimation error MS [m a.s.l.]]^2	Root of mean square estimation error RMSE [m a.s.l.]	Standard deviation of kriging $\sigma_{[\alpha]}$ [m a.s.l.]	Variance of estimation error [m a.s.l.]]^2	Mean of standardized cross--validation error	Variance of standardized cross--validation error	Coefficient of linear correlation r between Z i Z^*
a	-0.025499	104.580894	10.226480	10.580244	104.580244	-0.002494	1.000006	0,885
b	0.063270	72.518652	8.515788	72.514649	72.514649	0.007430	1.000006	0,920
c	0.055100	74.016182	8.603266	74.013146	74.013146	0.006405	1.000041	0,919
d	0.049184	75.191711	8.671315	75.189292	75.189292	0.005672	1.000032	0,917
e	0.050173	74.746623	8.645613	74.744105	74.744105	0.005803	1.000034	0,918

The choice of the theoretical semivariogram model used in the kriging estimation was based on the principle that the chosen model should lead to the smallest difference between the real values Z (original data) and the mean estimated values Z^* (Namysłowska-Wilczyńska, 2006). Consequently, the most important were two indices: mean of cross-validation errors (ME) and root of mean of square cross-validation errors (RMSE). This semivariogram model was chosen, which resulted in their smallest values.

During the validation of the theoretical semivariogram models the cross-validation error was referred to the standard deviation of kriging ($\sigma_{[\alpha]}$), which represents the assumed error, in order to compare the real and assumed error (Issaks and Srivastava, 1989, Namysłowska-Wilczyńska, 2006):

$$\frac{Z(x_\alpha) - Z^*(x_{[\alpha]})}{\sigma_{[\alpha]}}$$

where:

$Z(x_\alpha)$ – original value

$Z^*(_{[\alpha]})$ – mean estimated value at location x_α calculated after the elimination of the original value,

$\sigma_{[\alpha]}$ – standard deviation of kriging.

Also the mean of square standard errors of cross-validation (variance of standardised error) was calculated (Namysłowska-Wilczyńska, 2006):

$$\frac{1}{n}\sum_{\alpha=1}^{n} \frac{(Z(x_\alpha) - Z^*(x_{[\alpha]}))^2}{\sigma 2_{[\alpha]}}$$

This index gives an insight into the adequacy of the theoretical model that approximates the course of the experimental semivariogram (Wackernagel, 2003). If the true error of estimation were on average equal to the assumed error, than the value of the index would be 1.

The quality of interpolation was estimated basing on statistical parameters of the set of estimated values in all nodes of the elementary grid (Table 5) and on the estimate of cartographic correctness of the digital elevation models (Fig. 7). Mean error of estimation was used as a criterion of the estimation:

$$Z - Z_{grd}$$

where:

Z – value measured in sample point (Z)

Z_{grd} – value interpolated in point Z

as well as the estimate of variance (s^2) and standard deviation (s) of the interpolation error.

Table 5. Statistical parametres of the set of estimated values in all nodes of grid.
Tabela 5. Parametry statystyczne zbioru wartości estymowanych we wszystkich węzłach siatki.

Interpolation with semivariance model	Mean [m]	Min. [m]	Median [m]	Max. [m]	Standard deviation [m]	Variance $[m]^2$	Variability coefficient	Coefficient of skewness
a	256.7020	197.68	258.3865	328.62	23.2601	541.0311	0.091	0,035
b	254.9486	182.77	257.3212	344.00	26.3190	692.6891	0.103	-0,121
c	255.1060	184.05	257.2364	342.19	26.0596	679.1013	0.102	-0,108
d	255.2074	185.17	257.2394	341.42	25.8778	669.6602	0.101	-0,098
e	255.1882	184.96	257.2425	341.67	25.9248	672.1947	0.102	-0,101

Taking into account the magnitude of the nugget variance and the results of cross-validation, the choice of a theoretical variability model was limited to models c and b. Model c (nugget + exponential + power) features the lowest nugget variance. Model b (nugget + power) has the lowest values of: root of mean square error of estimation (RMSE), standard deviation of kriging ($\sigma_{[\alpha]}$), and variance of estimation error. This allows to accept the estimation using model b as the most accurate. Consequently, this model was chosen for the estimation of the values of elevation of the sub-Quaternary surface.

The choice of model b is also supported by the highest value of the linear correlation coefficient between the sampled and the estimated values. The adequacy of the choice of model b is also confirmed by the variance of the standardized cross-validation error, closest to 1 among the validated models. The mean of cross-validation errors close to zero indicates that there is no systematic overestimation or underestimation of the estimated values.

The mean of the values estimated using model b (Table 6) is lower by 4.48 m than the mean sample. The estimated minimum value exceeds the minimum sample value by 30.76 m. The maximum interpolated value is lower by 15.4 m than the lowest sample value. The distribution was thus „flattened", especially with respect to the minimum value. The „flattening" of distributions of the estimated values is greater when the other models are used.

Table 6. Selected statistical parameters of the set of estimated values *Z of the sub-Pleistocene surface elevation**
Tabela 6. Wybrane parametry statystyczne zbioru wartości estymowanych *Z wysokości bezwzględnej powierzchni podplejstoceńskiej**

Estimation with semivariance model	Estimated value Z* [m a.s.l.]				Standard deviation Z* [m]	Variance Z* [m]^2
	Min.	Median	Max.	Mean		
a	210.09	258.81	328.57	259.41	17.71	313.84
b	197.34	259.37	341.43	259.49	19.41	376.68
c	198.70	259.32	340.76	259.49	19.25	370.64
d	199.53	259.25	340.28	259.48	19.15	366.70
e	199.44	259.26	340.35	259.48	19.18	367.97

The digital models (Fig. 6) were constructed using as a framework an elementary grid consisting of 6319 nodes distributed every 500 m. The mean error of interpolation equalled 0.231 m in all cases.

The variation of parameters of the sets of estimated values in all grid nodes is rather small, especially with respect to the estimation using models b, c, d and e (Table 5). Also the digital elevation models interpolated using them are similar in general outlines and in details.

A conclusion drawn from the evaluation of the quality of interpolation estimation is that only hypsometry interpolated using model a (Fig. 6a) differs essentially from those obtained using the other models. This is related to the model's markedly higher nugget variance.

Anisotropy of the elevation values of the sub-Quaternary surface

The first part of the geostatistical analysis was done using isotropic variograms. At the angular tolerance level of 90^0 they take into account the maximum number of sample point pairs in each class and their course is the most „smoothed". According to B. Namysłowska-Wilczyńska (2006) an isotropic averaged semivariogram provides the best estimate of the nugget effect *(C_0)* and sill variance *(C')*, it also facilitates the choice of the theoretical model that should „fit" the experimental curve.

The second stage of the analysis consisted in construction of a directional semivariogram. The initial study of anisotropy was made at experimentally established angular tolerance of the semivariogram of 40º, in four directions: 0º, 45º, 90º and 135º (Fig. 7a–d).

The elevation of the sub-Quaternary surface shows the best spatial correlation at azimuth 147^0, semivariogram tolerance 40^0 and anisotropy coefficient 2 (Fig. 7e); the nugget variance equals 44.9 m^2, sill variance of the theoretical model – 0.768 m^2, the exponent of the power function 0.635, range 1.

Interpolation of the elevation of the sub-Quaternary surface

The choice of ordinary point kriging was dictated by the author's earlier experience in the application of computer-assisted methods of spatial data analysis (Szubert, 2004) and the other assesments of the ordinary kriging (Issaks and Srivastava, 1989, Wackernagel, 2003, Namysłowska-Wilczyńska, 2006). This method:

– belongs to the most frequently used kriging estimators,

– is a „strong" kriging system, accounting for the strong skewness of data distribution,

– when compared with other methods of interpolation it works well with unevenly distributed data; point

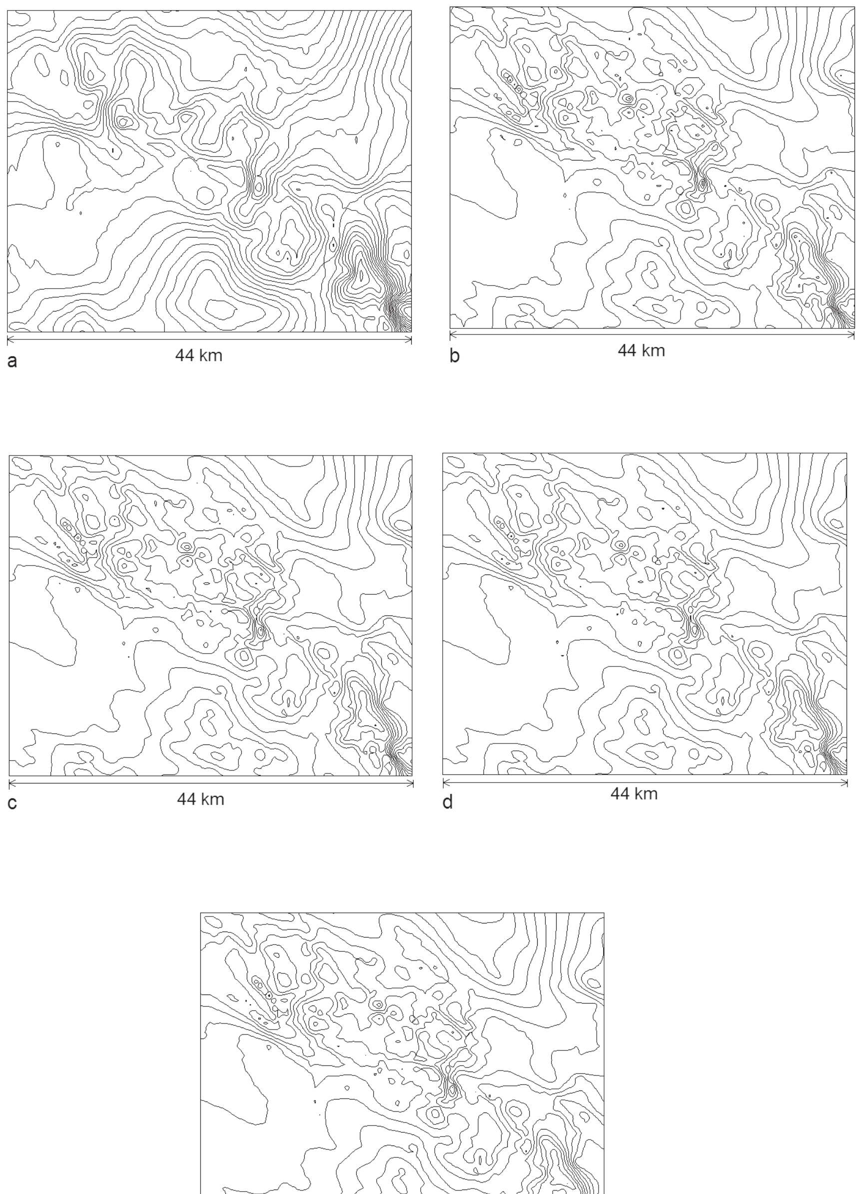


Fig. 6. Digital elevation models of the sub-Quaternary surface interpreted using various geostatistical models of data variability a – interpolation with model a (nugget + linear), b – interpolation with model b (nugget + power), c – interpolation with model c (nugget + exponential + power), d – interpolation with model d (nugget + exponential + power), e – interpolation with model e (nugget + pentaspherical + power).

Ryc. 6. Cyfrowe modele wysokościowe powierzchni podczwartorzędowej interpolowane z zastosowaniem różnych modeli geostatystycznych zmienności danych a – interpolacja z modelem a (nugget effect, linear), b – interpolacja z modelem b (nugget effect power), c – interpolacja z modelem c (nugget effect, exponential, power), d – interpolacja a modelem d (nugget effect, exponential, power), e – interpolacja z modelem e (nugget effect, pentaspherical, power).

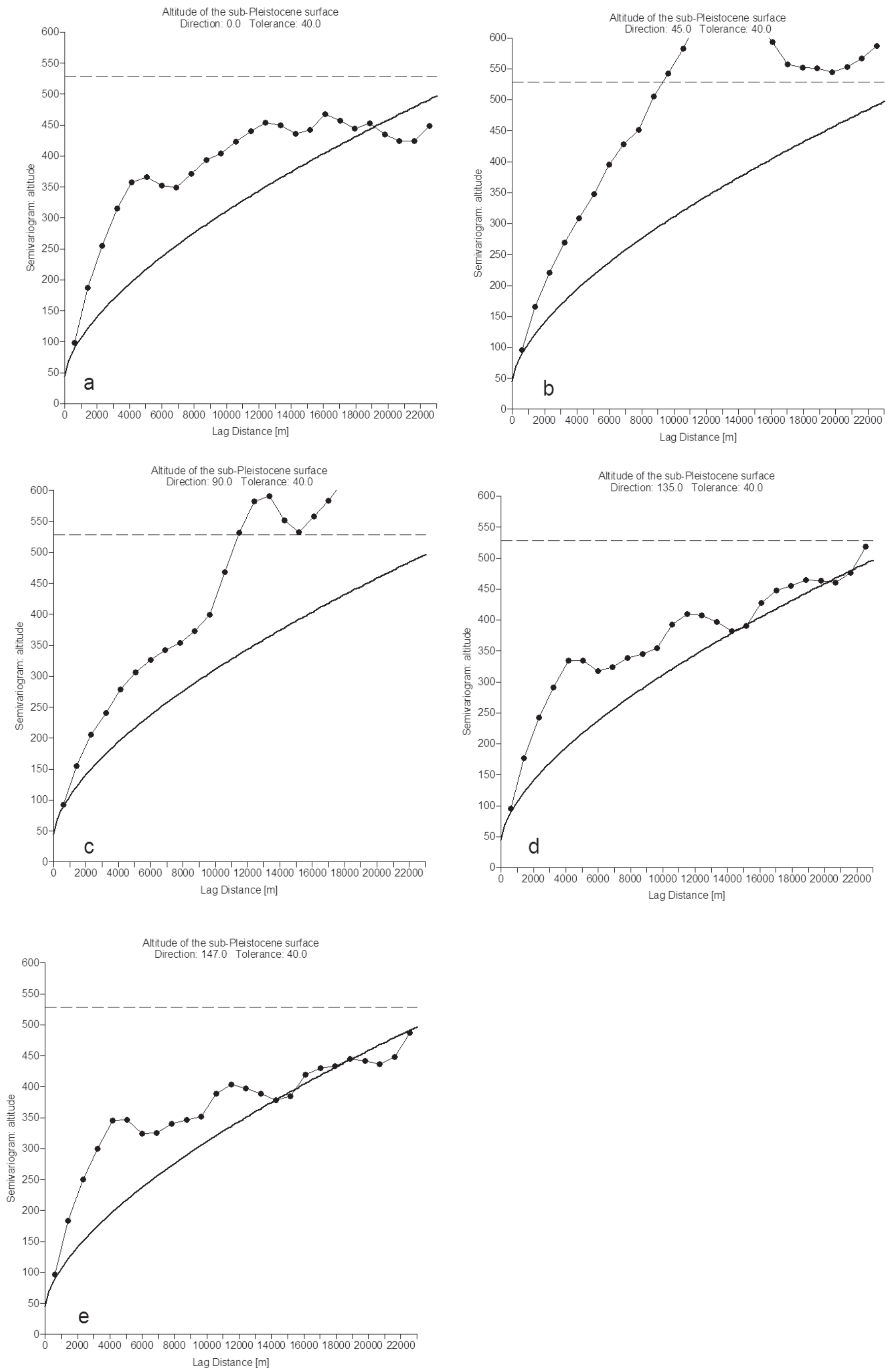


Fig. 7. Directional semivariograms of the elevation of the sub-Pleistocene surface. Angular tolerance of semivariograms 40^0. Azimuths: a – 0^0, b – 45^0, c – 90^0, d – 135^0, e – 147^0.

Ryc. 7. Semiwariogramy kierunkowe wysokości bezwzględnej powierzchni podplejstoceńskiej. Tolerancja kątowa semiwariogramów 40^0. Kierunki: a – 0^0, b – 45^0, c – 90^0, d – 135^0, e – 147^0.

concentration less influences the results of estimation than in other methods,

– it is a precise estimator, as is shown by the concordance between the estimated value *(Z*)* and data *(Z)*, when an estimated point (x_0) coincides with a sample point (x_α).

Moreover, estimation using ordinary kriging is based on a statistical description of data continuity by using a theoretical semivariogram model. The influence of the distances of the measurement points from the estimation point on the estimated mean *(Z*)* is lower than in other methods of spatial data analysis. Calculation errors are close to 0 and are distributed more evenly than in other kriging systems.

Geometry of the elementary grid and parameters of the area of point search were determined before performing the final interpolation of the sub-Quaternary surface elevation. Anisotropy of the data was reflected in the elliptical shape of the data area with semiaxes lengths R_1 (0X) and R_2 (0Y) equal to 35 000 m and 17 500 m, respectively, and semiaxis R_1 sloping toward 147º.

Another test performed was that of the accuracy of interpolation at various densities of the elementary grid and at various numbers of subsectors in the area searched for sample values (Table 7).

It was found that the value of error, hence the accuracy of interpolation, depends neither on the number of sectors in the area searched for sample values nor on the maximum and minimum numbers of sample points taken into account in the estimation of *Z** value, but rather on the distance between the nodes of the elementary grid (grid density).

Table 7. Mean errors of interpolation of the relief of the sub-Quaternary surface
Tabela 7. Średnie błędy interpolacji hipsometrii powierzchni podczwartorzędowej

Parameters of the search area	Distances between the nodes of elementary grid Number of nodes in elementary grid Mean error of interpolation [m] Variance of interpolation error (s^2) Standard deviation of the interpolation error (s)			
	100	250	500	1000
	154,791	24,957	6,319	1620
Number of sectors: **4** Maximum number of data introduced from all sectors: **64** Minimum number of data introduced from all sectors: **16**	0.0296 s^2 = 33.506 s = 5.79	0.0687 s^2 = 43.717 s = 6.612	0.231 s^2 = 56.547 s = 7.520	0.996 s^2 = 85.028 s = 9.221
Number of sectors: **8** Maximum number of data introduced from all sectors: **256** Minimum number of data introduced from all sectors: **32**	0,0229 s^2 = 33.506 s = 5.79	0,0625 s^2 = 43.708 s = 6.611	0,223 s^2 = 56.517 s = 7.518	0,983 s^2 = 84.736 s = 9.205
Number of sectors: **12** Maximum number of data introduced from all sectors: **600** Minimum number of data introduced from all sectors: **50**	0.0214 s^2 = 33.506 s = 5.79	0.0607 s^2 = 43.711 s = 6.611	0.221 s^2 = 56.520 s = 7.518	0.980 s^2 = 84.741 s = 9.205

Additionally, the quality of the interpretation was checked based on the cartographic correctness of the palaeoelevation contours (e.g. lack of loops, intersections, sharp bends). The mean error of interpolation does not exceed 1 m in all cases, which allows to refine the elevation model by using a narrower contour interval (Table 7).

The accuracy of the interpolation increased with increasing density of the elementary grid. The smallest interpolation error was obtained for a grid with 100 m distance between the nodes. The accuracy increased also with the increasing number of subsectors in the data search area. It was greatest at 12 subsectors and at the maximum number of introduced data, that is 600. Similar relations exist for data variability. It was lowest and independent of the number of data for the densest grid.

Cartographic correctness of the contour pattern depended on both grid density (Fig. 8a-d) and – to a lesser degree – the number of data (Fig. 8a, 8e-f). The model finally chosen was one interpolated on the basis of an elementary grid with 100 m node distance, estimated using the subdivision of the data searching area into 8 subsectors (Fig. 9 a).

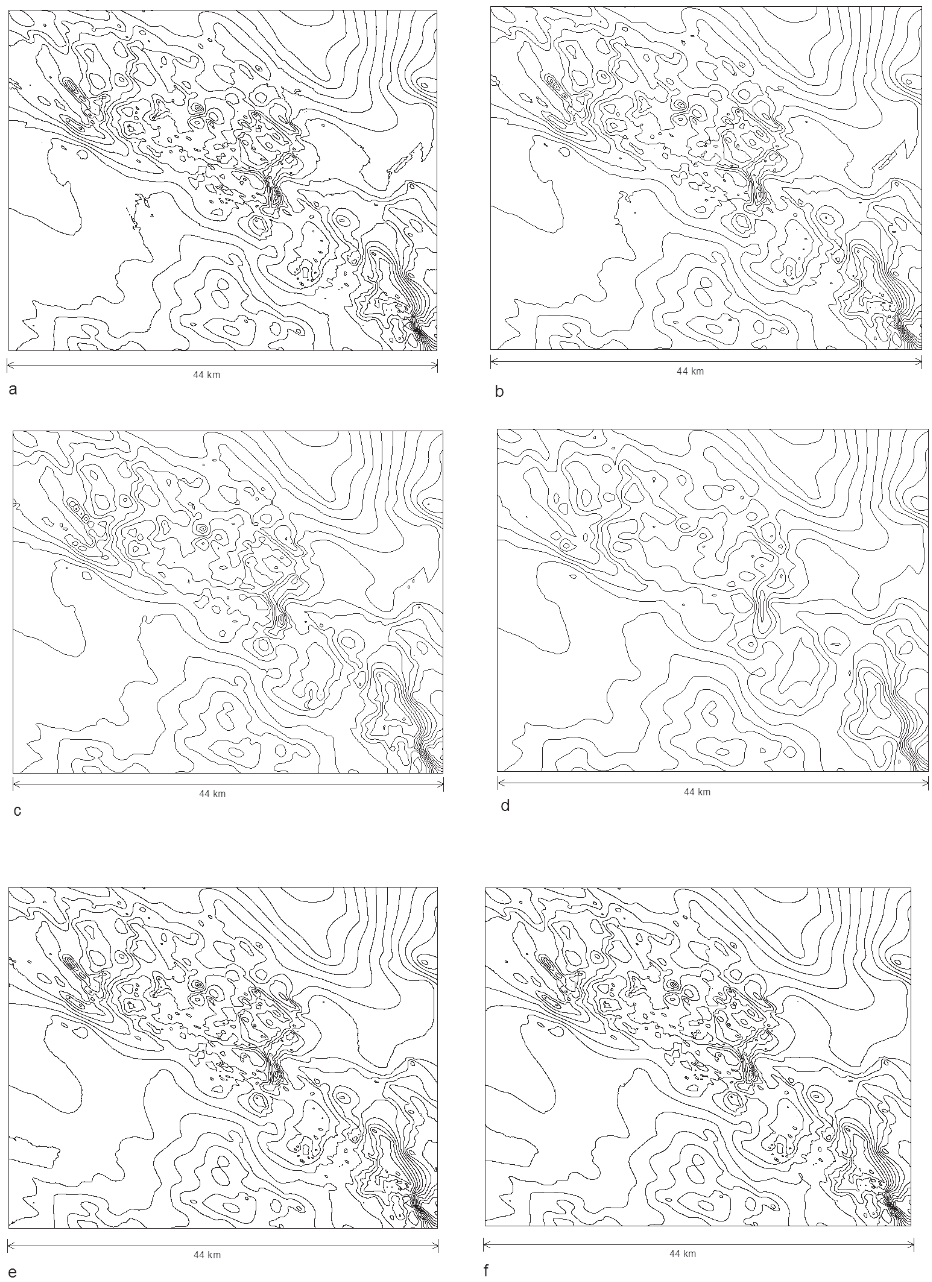


Fig. 8. Interpolation of palaeoelevation contours of the sub-Pleistocene surface: a-d − at 4 subsectors depending on the density of elementary grid, a – internodal distance of 100 m, b – internodal distance of 250, c – internodal distance of 500 m, d – internodal distance of 1000 m; e-f depending on the number of subsectors at internodal distances of 100 m, e – 8 subsectors, f – 12 subsectors.

Ryc. 8. Interpolacja paleoizohips powierzchni podplejstoceńskiej: a-d przy 4 podsektorach w zależności od gęstości siatki elementarnej a – odległości pomiędzy węzłami 100 m, b – odległości pomiędzy węzłami 250, c – odległości pomiędzy węzłami 500 m, d – odległości pomiędzy węzłami 1000 m; e-f w zależności od liczby podsektorów przy odległości pomiędzy węzłami siatki elementarnej 100 m e – 8 podsektorów, f – 12 podsektorów.

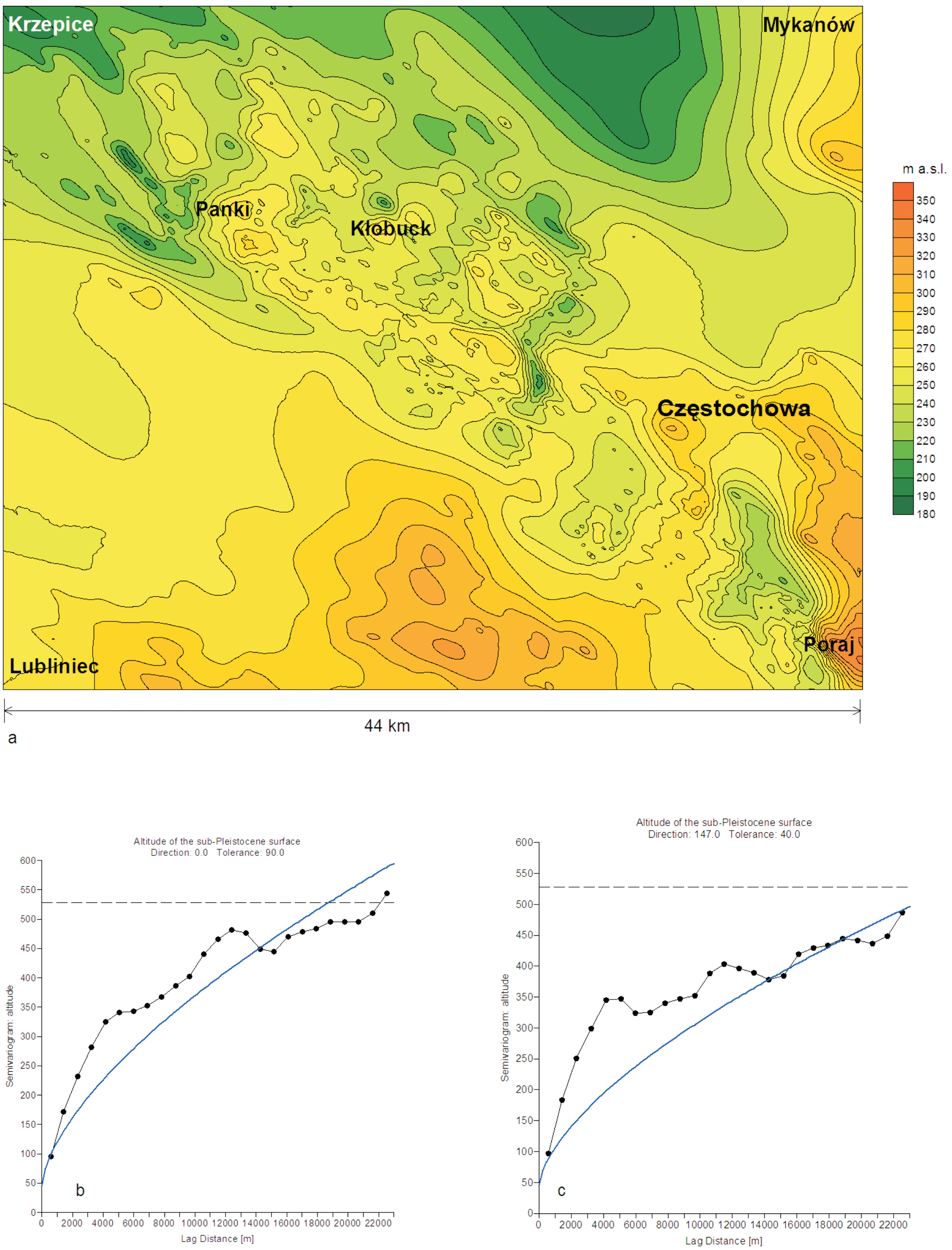


Fig. 9. Digital elevation model of the sub-Quaternary surface
a – digital elevation model, b – multidirectional geostatistical model, c –directional geostatistical model.
Ryc. 9. Cyfrowy model wysokościowy powierzchni podczwartorzędowej
a – cyfrowy model wysokościowy, b – wielokierunkowy model geostatystyczny, c – kierunkowy model geostatystyczny.

Discussion of results

The results of this study are visualized as a juxtaposition of the digital elevation model with an isotropic and a directional semivariograms. The spatial structure of the data displays a fluctuation of variable amplitude and drift. The spatial correlation of the data is disturbed. This may be interpreted as a manifestation of a local activity of one or several relief-forming factors that distorted the large-scale (prevailing over the whole area) features of the elevation pattern, shaped by a long-lasting geomorphological process. The spatial order (better correlation of data) is present in the NNE-SSW direction.

The last stage of shaping the sub-Pleistocene surface on the Woźniki-Wieluń Upland took place during the Odra Glaciation. It was then when the factor was active and distorted the relief characteristics shaped earlier, during the Mazovian Interglacial.

The Mazovian Interglacial was a period of a long-lasting denudation (Mojski, 2005), when the valley network was formed. The valleys had smooth long profiles. The smoothing of relief was interrupted by the invasion of the Odra ice sheet. Subglacial waters shaped troughs and basins, sharply marked in palaeorelief. The fluctuation in the spatial structure of data may record subglacial erosion. Subglacial troughs were an element of the valley network draining the subglacial waters during the pre-Warta interstadial.

Another stage of intense denudation followed during the Warta Stadial, when the northern part of the Upland was covered with an ice sheet. The studied area lay at its foreland. Periglacial climate intensified denudation of the depositional forms formed during the decay of the Odra ice sheet. Erosional subglacial forms were filled with periglacial sediments and so they were preserved.

This discussion is an attempt at explaining the fluctuation and drift basing on the previous knowledge on the palaeogeographical evolution of the northern part of the Kraków-Silesia Upland.

The digital elevation model (Fig. 9a) revealed a series of depressions in the sub-Pleistocene surface between Poraj and Krzepice. The performed validation of estimation and interpolation of the elevation of the sub-Pleistocene surface demonstrates that the pattern of these depressions, their shapes and sizes closely reflect the true relief.

The depressions south of Częstochowa were described by Z. Mossoczy (1955), who postulated that they are fragments of the pre-glacial valley of the upper Warta River (Figs 10, 11); a hypothesis accepted also in later works (Klimek, 1961, 1966). Similar depressions, also interpreted as fragments of buried valleys, are present in the northern part of the Woźniki-Wieluń Upland (Krzemiński, 1974) and west of Częstochowa (Lewandowski, 1993) (Figs 12, 13).

The digital elevation model (Fig. 9a) shows three separate groups of depressions: near Częstochowa, Kłobuck and Krzepice. They are incised into a plateau sloping to the northwest and north, toward an extensive depression.

The depressions near Częstochowa are clearly separated (Fig. 14c) by a ridge culminating in the Jasna Góra hill. They are shaped like wide bowls. The shallower one (30–35 m deep) is ovate, the deeper one (50–60 m) is elongated meridionally. They are probably a part of a greater system of depressions, as is shown by a trough branching to the south. Ovate and elongate secondary depressions are present in their bottoms.

One of the depressions, situated west of Częstochowa, resembles in its shape a classical subglacial trough (Fig. 14b). It consists of three parts. On the south it terminates in a bowl-shaped hollow with a radius of 2–2.5 km, ca. 35 m deep. The middle, meridionally elongated part of the trough is deepest (up to 70 m) and narrowest (1–2 km in the upper part). Secondary depressions are incised in the bottom. The third, northern part is up to 50 m deep and elongated in the NW-SE direction.

The depressions near Krzepice (Fig. 10a) are interconnected troughs with sharply outlined ovate or elongated hollows up to 50 m deep. The troughs are sloping to the northwest and north. Broad valleys with inner depressions in shape of subglacial bowls are present near Kłobuck (Fig. 9a).

The analysis of the sub-Pleistocene surface hypsometry does not support a thesis that the buried depressions between Poraj and Krzepice on the Wieluń Upland are fragments of the palaeo-Warta valley. The shapes and sizes of these forms support the thesis formulated by J. Lewandowski (1993) who, based on the lithological differentiation of the sediments filling some of the deep buried depressions, suggested that these are subglacial troughs or exaration depressions.

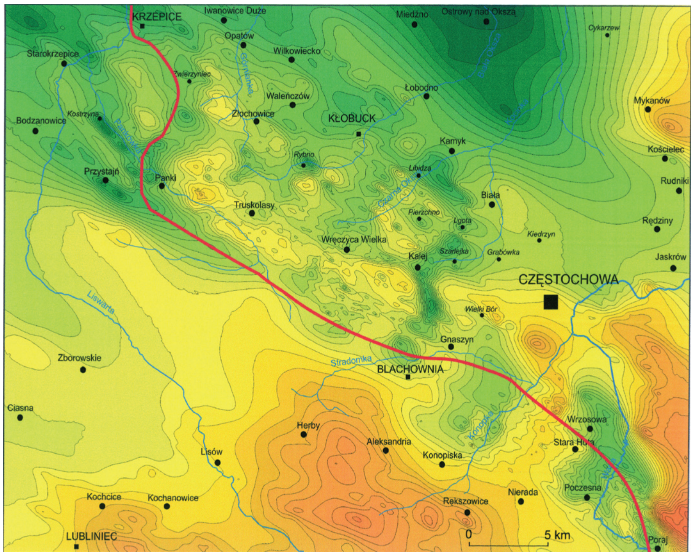


Fig. 10. Pre-glacial valley of the palaeo-Warta River after Z. Mossoczy (1955) against the digital elevation model of the sub--Pleistocene surface.

Ryc. 10. Preglacjalna dolina pra-Warty wg Z. Mossoczego (1955) na tle cyfrowego modelu wysokościowego powierzchni podplejstoceńskiej.

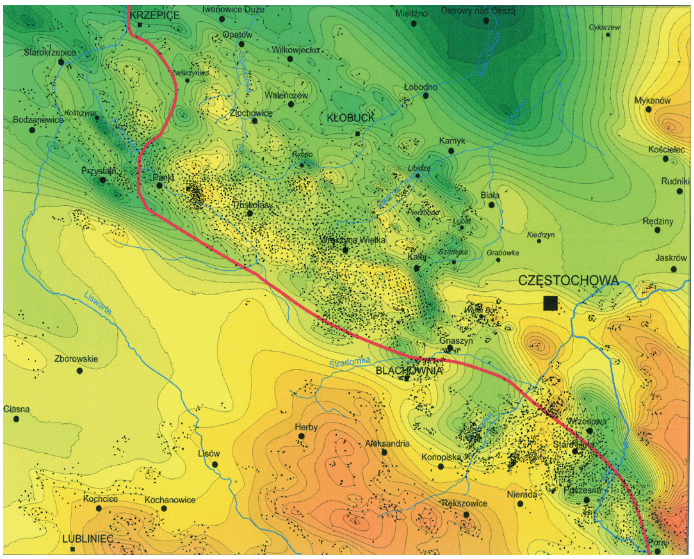


Fig. 11. Pre-glacial valley of the palaeo Warta River after Z. Mossoczy (1955) against the sample points used for construction of the digital elevation model of the sub-Pleistocene surface.

Ryc. 11. Preglacjalna dolina pra-Warty wg Z. Mossoczego (1955) na tle punktów próbkowych użytych do opracowania cyfrowego modelu wysokościowego powierzchni podplejstoceńskiej.

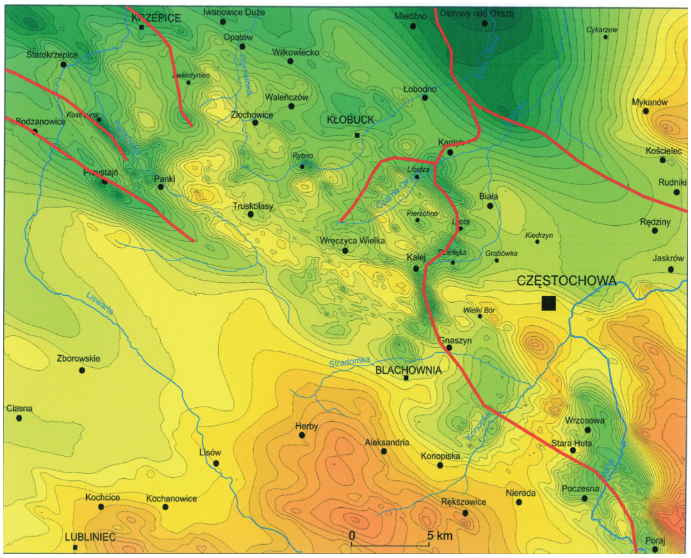


Fig. 12. Pre-glacial valley network after J. Lewandowski (1993) against the digital elevation model of the sub-Pleistocene surface.

Ryc. 12. Preglacjalna sieć dolinna wg J. Lewandowskiego (1993) na tle cyfrowego modelu wysokościowego powierzchni podplejstoceńskiej.

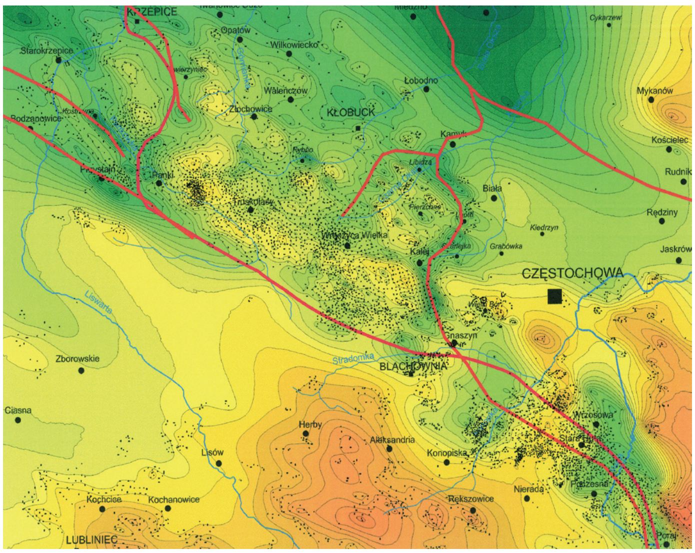


Fig. 13. Pre-glacial valley network after J. Lewandowski (1993) against the sample points used for the construction of the digital elevation model of the sub-Pleistocene surface.

Ryc. 13. Preglacjalna sieć dolinna wg. J. Lewandowskiego (1993) na tle punktów próbkowych użytych do opracowania cyfrowego modelu wysokościowego powierzchni podplejstoceńskiej.

The paper quoted above includes a palaeogeomorphological sketch of the sub-Pleistocene surface in the Silesia-Kraków region. A comparison of the Eopleistocene valley network in this scheme with the interpolated elevation model shows discrepancies (Fig. 12), which suggest that the depressions on the Woźniki-Wieluń Upland are not fragments of the Eopleistocene fluvial network. This conclusion is also supported by the pilot studies of the lithological differentiation of the sedimentary fill (Szubert, 2008a,b).

The distinguished depressions (Fig. 14a-c) should be considered as subglacial troughs formed during the Odra Glaciation – the last in this area. During the pre-Warta interstadial they were a part of the valley network.

The methodical aspect of the present paper provides an opportunity for reflection on the benefits of using for geomorphological purposes the geostatistical methods that seem to be difficult, complex and time-consuming when compared with other methods. A partial response was given in the paragraph explaining the choice of ordinary kriging as the method of interpolation.

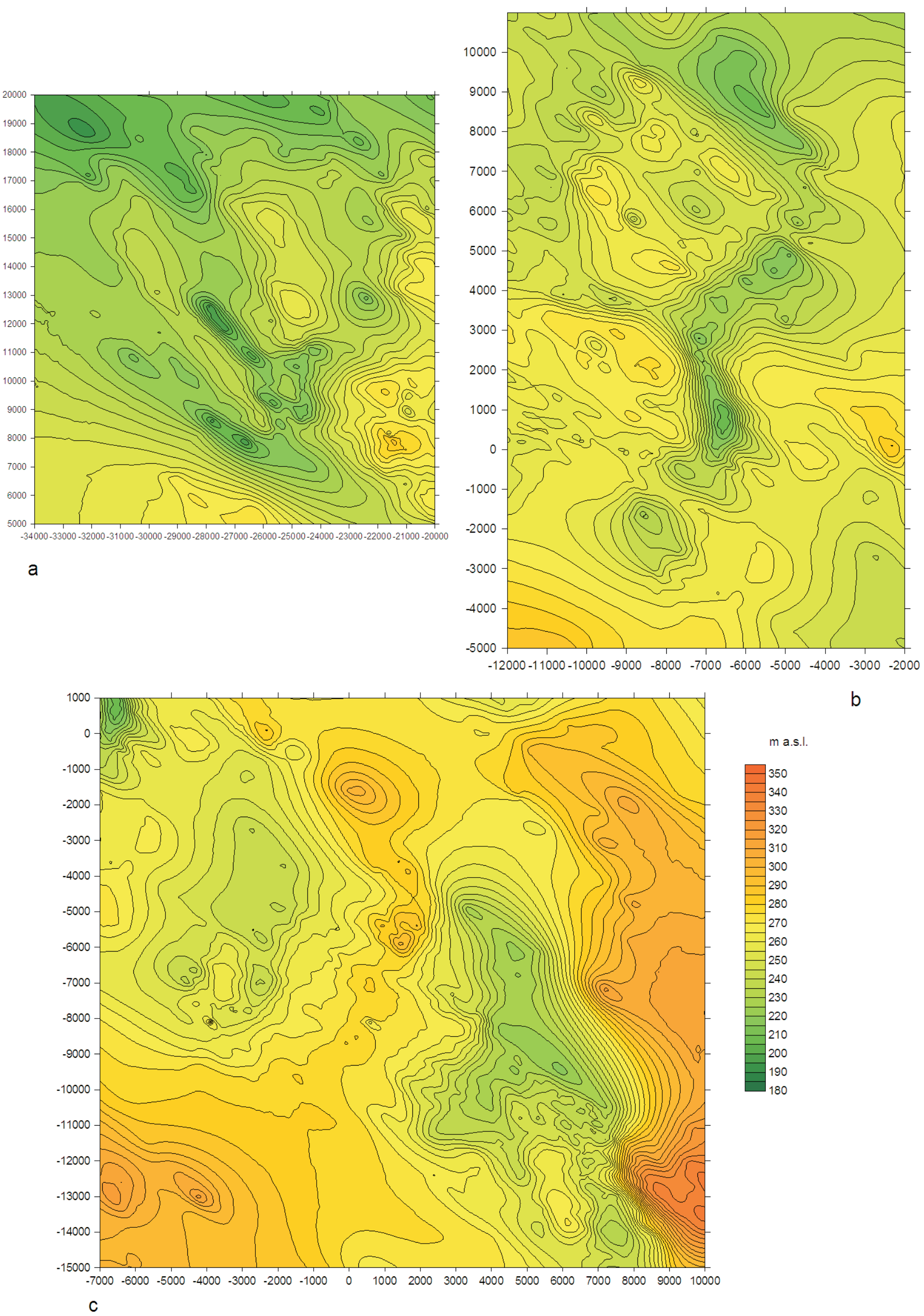
The advantages of geostatistical methods include:

– interpretation of the randomness of the phenomenon as fluctuation around drift, and fluctuations as vanishing properties of the phenomenon rather then errors, and acceptation of the drift as a reference surface for the fluctuations (Wackernagel, 2003). Multifactoriality of geomorphological phenomena and processes makes randomness their fundamental characteristic;

– structural analysis which allows to show in a semivariogram the spatial structure of regionalized data (studied phenomenon);

– data used in geomorphology are regionalized data. A analysis of their spatial structure may allow estimating the relative role of both, the structural and random components, showing to what degree they are correlated and to what degree chaotic. In geomorphology, it is the tendency to terrain planation, shaping of valleys with uniform slope. If a factor (one or more) disturbing denudation is (are) present, then structural analysis will reveal fluctuations and drift. It may be thus inferred to what degree is the analyzed hypsometry (relief) the result of local (random) processes. The structural analysis, by revealing the characteristics of the phenomenon which are not apparent in the hypsometrical pattern, is an important complement of a digital elevation model (Fig.9);

– possibility of using large data bases, conducting spatial analysis over a large area (interpolation and extrapolation of contour lines) and the valuation of the correctness of estimation and interpolation – judging from the accuracy of a digital model as a measure of its credibility. It is especially important when the object of study is not available for direct observation. Another conclusion stems from the comparison of Figures 10, 12 and 14. They show how the views on the pattern of buried valley network have evolved during the last 50 years with the growing data base, that is the knowledge on the relief of the buried surface and the development of interpolation methods from manual ones to those computer-assisted. A application of the latter, strengthened with geostatistical spatial analysis, allowed constructing a large-size credible image of palaeohypsometry, which led to the reinterpretation of the interpolated. In the case of the Woźniki-Wieluń Upland the geostatistical analysis indicated also a new dimension in the studies on buried relief – the evolution of this relief during the pre-Warta interstadial and subglacial erosion during the Odra Glaciation as important factors in shaping the valley network in the northern part of the Silesia-Kraków Upland.



Ryc. 14. Buried depressions on the Woźniki-Wieluń Upland
a – between Panki and Krzepice, b – between Konopiska and Czarny Las (west from Częstochowa), c – between Poraj and Częstochowa.

Ryc. 14. Kopalne obniżenia na Wyżynie Woźnicko-Wieluńskiej
a – pomiędzy Pankami i Krzepicami, b– pomiędzy Konopiskami i Czarnym Lasem (na zachód od Częstochowy), c – pomiędzy Porajem i Częstochową.

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Streszczenie

W publikacji przedstawiono wyniki badania hipsometrii podłoża plejstocenu metodą geostatystyczną na Wyżynie Woźnicko-Wieluńskiej. Głównym problemem badawczym było rozstrzygnięcie, czy głębokie, kopalne obniżenia są fragmentami preglacjalnej doliny pra-Warty *(Mossoczy, 1955, Klimek, 1961, 1966)*.

Paleohipsometrię zrekonstruowano na podstawie 6499 punktów (4862 odwierty, 1637 punkty na Szczegółowej Mapie Geologicznej Polski) w południowej części Wyżyny Woźnicko-Wieluńskiej, na obszarze 1540 km^2 (ryc. 1, ryc. 2). W pierwszej części opracowania przeprowadzono analizę geostatystyczną danych, a w drugiej – metodą krigingu zwyczajnego punktowego wyinterpolowano wysokość bezwzględną podłoża plejstocenu i zobrazowano ją za pomocą cyfrowego modelu wysokościowego.

Analiza geostatystyczna ujawniła strukturę przestrzenną danych, której cechy odzwierciedla krzywa izotropowego semiwariogramu empirycznego (ryc. 3). Semiwariogram empiryczny jest nieograniczony oraz nie ma progu, co wskazuje na istnienie trendu lub dryftu w zbiorze danych. Krzywa empiryczna nie jest wygładzona, wykazuje fluktuacje. Cechami struktury przestrzennej wysokości bezwzględnej powierzchni podplejstoceńskiej są: wariancja nuggetowa, wyraźna korelacja przestrzenna danych na którą nakłada się stabilizacja wartości funkcji semiwariogramu oraz odwrócenie tendencji wzrostowej semiwariogramu.

Krzywa empiryczna wskazuje na znaczący udział czynnika losowego w kształtowaniu hipsometrii podłoża plejstocenu. Wyrazem tego są fluktuacje wartości semiwariancji oraz dryft w zbiorze danych. Stąd wniosek, że hipsometria prezentowana na cyfrowym modelu wysokościowym (ryc. 9) jest wynikiem działania procesu lokalnego (przypadkowego). Z uwagi na wyrazistość form erozyjnych takim procesem była erozja subglacjalna.

Krzywa empiryczna została przybliżona złożonymi modelami teoretycznymi (ryc. 5, tab. 3), z których na podstawie wyników kross-walidacji (tab. 4) oraz oceny interpolacji (ryc. 6, tab. 4) wybrano najlepiej „pasujący” do modelu empirycznego, zbudowanym z trzech modeli podstawowych: nugget effect, pentaspherical i power (ryc. 5e). Po zbadaniu anizotropii (ryc. 7) opracowano geostatystyczny model anizotropowy (ryc. 7e).

W powierzchni podplejstoceńskiej istnieją rynny oraz owalne zagłębienia o głębokości dochodzącej do 60 m. Grupują się one w okolicach Częstochowy (ryc. 14b, c), pomiędzy Pankami i Krzepicami (ryc. 14a) oraz w rejonie Kłobucka (ryc. 9a). Z uwagi na ich kształt oraz głębokość należałoby je uznać za rynny oraz misy subglacjalne, co potwierdza hipotezę postawioną przez *J. Lewandowskiego (1993)*. Ich geneza wiązałaby się ze zlodowaceniem odry– ostatnim na badanym terenie. Formy te wypełnione są zróżnicowanymi litologicznie osadami *(Szubert, 2004, 2008a, b)*, co wskazuje, że w schyłkowej fazie zlodowacenia odry oraz interstadiale przedwarciańskim były one elementem sieci dolinnej.

W czasie arealnej deglacjacji lądolodu odry poprzez obniżenia subglacjalne odpływały wody proglacjalne. Niektóre były jeziorami. W interstadiale przedwarciańskim rynny w okolicach Panek i Krzepic oraz Kłobucka zostały włączone w sieć dolinną odwadniającą południową część Wyżyny na północ i północny zachód. W czasie stadiału warty doliny wypełnione zostały osadami peryglacjalnymi. Badany teren znajdował się wówczas na przedpolu lądolodu, co sprzyjało intensywnej denudacji peryglacjalnej.

Ryc. 10, 12 i 14 ukazują rozwój poglądów w ciągu 50 lat na układ kopalnej sieci dolinnej, w sytuacji powiększania bazy danych, a w związku z tym wiedzy o ukształtowaniu kopalnej powierzchni oraz rozwoju metod interpolacji od manualnych po komputerowe. Zastosowanie tych ostatnich, wzbogaconych o geostatystyczną analizę przestrzenną pozwoliło opracować wielkoprzestrzenny, wiarygodny obraz paleohipsometrii skłaniający do reinterpretacji poznanych wcześniej *(Mossoczy, 1955)* obniżeń. Przeprowadzone badania nie potwierdziły tezy, że kopalne zagłębienia są fragmentami preglacjalnej doliny pra-Warty (ryc. 10, 12).